

$$\begin{aligned} \boxed{1} \quad (1) \quad (x^3 + 4 - 3x)(1 - 2x) &= (x^3 + 4 - 3x) \cdot 1 + (x^3 + 4 - 3x) \cdot (-2x) \\ &= x^3 + 4 - 3x - 2x^4 - 8x + 6x^2 \\ &= -2x^4 + x^3 + 6x^2 - 11x + 4 \end{aligned}$$

$$\begin{aligned} (2) \quad (x - a)(x - b)(x - c) &= \{x^2 - (a + b)x + ab\}(x - c) \\ &= \{x^2 - (a + b)x + ab\} \cdot x + \{x^2 - (a + b)x + ab\} \cdot (-c) \\ &= x^3 - (a + b)x^2 + abx - cx^2 + (a + b)cx - abc \\ &= x^3 - (a + b + c)x^2 + (ab + bc + ca)x - abc \end{aligned}$$

$$\begin{aligned} (3) \quad (x^2 - x + 1)(x^2 - x + 3) &= (x^2 - x)^2 + 4(x^2 - x) + 3 \\ &= x^4 - 2x^3 + x^2 + 4x^2 - 4x + 3 \\ &= x^4 - 2x^3 + 5x^2 - 4x + 3 \end{aligned}$$

$$\begin{aligned} (4) \quad (x + 1)(x + 2)(x - 5)(x - 6) &= (x + 1)(x - 5) \times (x + 2)(x - 6) \\ &= (x^2 - 4x - 5)(x^2 - 4x - 12) \\ &= (x^2 - 4x)^2 - 17(x^2 - 4x) + 60 \\ &= x^4 - 8x^3 + 16x^2 - 17x^2 + 68x + 60 \\ &= x^4 - 8x^3 - x^2 + 68x + 60 \end{aligned}$$

$$\boxed{2} \quad (1) \quad 6x^2 + (3a - 2b)x - ab = (2x + a)(3x - b)$$

$$\begin{aligned} (2) \quad x^2 + y^2 - 2xy - z^2 &= (x - y)^2 - z^2 \\ &= \{(x - y) + z\}\{(x - y) - z\} \\ &= (x - y + z)(x - y - z) \end{aligned}$$

$$\begin{aligned} (3) \quad 3x^2 + ax - 2a^2 + 4x - a + 1 &= 3x^2 + (a + 4)x - (2a^2 + a - 1) \\ &= 3x^2 + (a + 4)x - (a + 1)(2a - 1) \\ &= \{x + (a + 1)\}\{3x - (2a - 1)\} \\ &= (x + a + 1)(3x - 2a + 1) \end{aligned}$$

$$\begin{aligned} (4) \quad ab^2 - bc^2 - b^2c - c^2a &= (b^2 - c^2)a - bc(b + c) \\ &= (b + c)(b - c)a - bc(b + c) \\ &= (b + c)\{(b - c)a - bc\} \\ &= (b + c)(ab - bc - ca) \end{aligned}$$

$$\begin{aligned} \boxed{3} \quad (1) \quad (1 + \sqrt{2} + \sqrt{3})^2 &= (1 + \sqrt{2})^2 + 2(1 + \sqrt{2})\sqrt{3} + (\sqrt{3})^2 \\ &= 1^2 + 2\sqrt{2} + (\sqrt{2})^2 + 2\sqrt{3} + 2\sqrt{2}\sqrt{3} + 3 \\ &= 6 + 2\sqrt{2} + 2\sqrt{3} + 2\sqrt{6} \end{aligned}$$

$$\begin{aligned} (2) \quad (\sqrt{2} + \sqrt{3} + \sqrt{5})(\sqrt{2} + \sqrt{3} - \sqrt{5}) &= (\sqrt{2} + \sqrt{3})^2 - (\sqrt{5})^2 \\ &= (\sqrt{2})^2 + 2\sqrt{2}\sqrt{3} + (\sqrt{3})^2 - 5 \\ &= 5 + 2\sqrt{6} - 5 \\ &= 2\sqrt{6} \end{aligned}$$

$$\begin{aligned}
 (3) \quad \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{8}} + \frac{1}{\sqrt{32}} &= \frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}} + \frac{1}{4\sqrt{2}} \\
 &= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{8} \\
 &= \left(\frac{1}{2} - \frac{1}{4} + \frac{1}{8}\right)\sqrt{2} \\
 &= \frac{3\sqrt{2}}{8}
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad \frac{2-\sqrt{3}}{2+\sqrt{3}} + \frac{2+\sqrt{3}}{2-\sqrt{3}} &= \frac{(2-\sqrt{3})^2}{(2+\sqrt{3})(2-\sqrt{3})} + \frac{(2+\sqrt{3})^2}{(2-\sqrt{3})(2+\sqrt{3})} \\
 &= \frac{2^2-4\sqrt{3}+(\sqrt{3})^2+2^2+4\sqrt{3}+(\sqrt{3})^2}{2^2-(\sqrt{3})^2} \\
 &= 4-4\sqrt{3}+3+4+4\sqrt{3}+3 \\
 &= 14
 \end{aligned}$$

$$\begin{aligned}
 (5) \quad \frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+2} \\
 &= \frac{1-\sqrt{2}}{(1+\sqrt{2})(1-\sqrt{2})} + \frac{\sqrt{2}-\sqrt{3}}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})} + \frac{\sqrt{3}-2}{(\sqrt{3}+2)(\sqrt{3}-2)} \\
 &= \frac{1-\sqrt{2}}{1-(\sqrt{2})^2} + \frac{\sqrt{2}-\sqrt{3}}{(\sqrt{2})^2-(\sqrt{3})^2} + \frac{\sqrt{3}-2}{(\sqrt{3})^2-2^2} \\
 &= \sqrt{2}-1+\sqrt{3}-\sqrt{2}+2-\sqrt{3} \\
 &= 1
 \end{aligned}$$

$$\boxed{4} \quad \sqrt{x^2-2x+1} = \sqrt{(x-1)^2} = |x-1|$$

$$(1) \quad x \geq 1 \text{ のとき, } x-1 \geq 0 \text{ であるから } \sqrt{x^2-2x+1} = x-1$$

$$(2) \quad x < 1 \text{ のとき, } x-1 < 0 \text{ であるから } \sqrt{x^2-2x+1} = -(x-1) = -x+1$$

$$\boxed{5} \quad \text{両辺に } 6 \text{ を掛けると } 6\left\{\frac{1}{2}(n+3) + \frac{1}{6}\right\} > 6\left\{\frac{1}{3}(4n-1)\right\}$$

$$\text{すなわち} \quad 3n+9+1 > 8n-2$$

$$\text{移項して整理すると} \quad -5n > -12$$

$$\text{よって} \quad n < \frac{12}{5} = 2.4$$

$$\text{不等式を満たす自然数 } n \text{ は} \quad n = 1, 2$$

$$\boxed{6} \quad \text{歩く道のりを } x \text{ m とすると}$$

$$32 \leq \frac{x}{80} + \frac{4000-x}{200} \leq 35$$

各辺に 400 を掛けると

$$12800 \leq 5x + 2(4000-x) \leq 14000$$

$$\text{よって} \quad 12800 \leq 3x + 8000 \leq 14000$$

$$\text{各辺から } 8000 \text{ を引くと} \quad 4800 \leq 3x \leq 6000$$

$$\text{各辺を } 3 \text{ で割ると} \quad 1600 \leq x \leq 2000$$

答 1600 m 以上 2000 m 以下

$$\boxed{7} \quad |4x+2| < 11 \text{ より} \quad -11 < 4x+2 < 11$$

$$-13 < 4x < 9$$

$$\text{よって} \quad -\frac{13}{4} < x < \frac{9}{4}$$

$-\frac{13}{4} = -3.25$, $\frac{9}{4} = 2.25$ であるから, 不等式を満たす整数 x の個数は 6 個